Ariketak 2023/12/18

1. $\int \frac{x^2+4}{(x+2)^2}\,dx = \ EBAU\ 2022 - ekaina$

2. $\int \frac{x^3+x+1}{x^2+1}\,dx =$

3. $\int \frac{\sqrt{x}}{\sqrt[3]{x}-1}\,dx =$ (351.orr 16.ariketa c)

4. $\int \frac{5}{100x^2+1}\,dx =$ (350.orr 7.ariketa b)

5. $\int \frac{-x^2+7x}{x^3-x^2-x+1}\,dx =$

6. $\int \frac{x}{\sqrt{1+3x^2}}\,dx =$

7. $\int e^{-x}\sin(2x)\,dx =$

8. $\int \frac{x^3-x+6}{x^2+5x+4}\,dx =$

9. $\int \frac{x^3+4x^2-10x+7}{x^2-7x-6}\,dx =$

10. $\int \frac{2x^2+18x+25}{x^3+3x^2-4}\,dx =$

11. $\int Ln\left(\frac{x+1}{x-1}\right)^x dx = \ **$

12. $\int \frac{1}{\sqrt{x}(1+\sqrt[4]{x})}\,dx =$

13. $\int \frac{1}{(\sqrt{x}+\sqrt[3]{x})}\,dx =$

5.ariketa

$$\int\frac{-x^2+7x}{x^3-x^2-x+1}dx=\int\left(\frac{3}{(x-1)^2}+\frac{1}{x-1}+\frac{-2}{x+1}\right)dx=3\int\frac{1}{(x-1)^2}dx+\int\frac{1}{x-1}dx-2\int\frac{1}{x+1}dx=$$

$$=-\frac{3}{x-1}+\ln|x-1|-2\ln|x+1|+k$$

6.ariketa

h) $$\int\frac{x}{\sqrt{1+3x^2}}dx=\frac{1}{6}\int\frac{1}{\sqrt{t}}dt=\frac{2}{6}\sqrt{t}+k=\frac{1}{3}\sqrt{1+3x^2}+k$$

$$\boxed{t=1+3x^2\rightarrow dt=6x\,dx\rightarrow x\,dx=\frac{1}{6}dt}$$

7.ariketa

c) $$I=\int e^{-x}sen2x\,dx=-e^{-x}\frac{\cos 2x}{2}-\frac{1}{2}\int e^{-x}\cos 2x\,dx=-e^{-x}\frac{\cos 2x}{2}-\frac{1}{2}\left(e^{-x}\frac{sen2x}{2}+\frac{1}{2}\int e^{-x}sen2x\,dx\right)=$$

$$u=e^{-x}\rightarrow du=-e^{-x}dx \qquad dv=sen2x\,dx\rightarrow v=\frac{-\cos}{}$$

$$u=e^{-x}\rightarrow du=-e^{-x}dx \qquad dv=\cos 2x\,dx\rightarrow v=\frac{sen2}{}$$

$$=-e^{-x}\frac{\cos 2x}{2}-e^{-x}\frac{sen2x}{4}-\frac{1}{4}\int e^{-x}sen2x\,dx=-e^{-x}\frac{\cos 2x}{2}-e^{-x}\frac{sen2x}{4}-\frac{1}{4}I$$

$$I=-e^{-x}\frac{\cos 2x}{2}-e^{-x}\frac{sen2x}{4}-\frac{1}{4}I\rightarrow\frac{5}{4}I=-e^{-x}\left(\frac{\cos 2x}{2}+\frac{sen2x}{4}\right)\rightarrow I=\frac{-e^{-x}}{5}(2\cos 2x+sen2x)+k$$

8.ariketa

d) $$\int\frac{x^3-x+6}{x^2+5x+4}dx=\int\left(x+\frac{2}{x+1}+\frac{18}{x+4}-5\right)dx=\frac{1}{2}x^2+2\ln|x+1|+18\ln|x+4|-5x+k$$

9.ariketa

a) $$\int\frac{x^3+4x^2-10x+7}{x^3-7x-6}dx=\int\left(1+\frac{2}{x-3}-\frac{5}{x+1}+\frac{7}{x+2}\right)dx=$$

$$=x+2\ln|x-3|-5\ln|x+1|+7\ln|x+2|+k$$

169. $\displaystyle\int \frac{2x^2 + 18x + 25}{x^3 + 3x^2 - 4}\,dx$

10.ariketa

Solución:

Se aplica el método de integración de funciones racionales.

Raíces del denominador:

$$x = 1 \text{ real simple.}$$
$$x = -2 \text{ real doble.}$$

La descomposición es:

$$\frac{5}{x-1} - \frac{3}{x+2} + \frac{1}{(x+2)^2}$$

La integral es:

$$5\,L\,|x-1| - 3\,L\,|x+2| - \frac{1}{x+2} + k$$

11.ariketa

b) $\displaystyle\int \ln\left(\frac{x+1}{x-1}\right)^x dx = \int x\ln\left(\frac{x+1}{x-1}\right)dx = \int x\ln(x+1)\,dx - \int x\ln(x-1)\,dx = \frac{x}{x+1} - \int\frac{1}{x+1}dx - \frac{x}{x-1} + \int\frac{1}{x-1}dx =$

$\boxed{\begin{array}{l} u = x \rightarrow du = dx \\ dv = \ln(x+1)dx \rightarrow v = \end{array}}$ $\boxed{\begin{array}{l} u = x \rightarrow du = dx \\ dv = \ln(x-1)dx \rightarrow v = \end{array}}$

12.ariketa

e) $\displaystyle\int \frac{1}{\sqrt{x}\left(1+\sqrt[4]{x}\right)}dx = \int\frac{1}{t^2(1+t)}\cdot 4t^3dt = 4\int\frac{t}{1+t}dt = 4\int\left(1+\frac{-1}{1+t}\right)dt = 4t - 4\ln|t+1| + k = 4\sqrt[4]{x} - 4\ln|\sqrt[4]{x}+1| + k$

$\boxed{t = \sqrt[4]{x} \rightarrow dt = \dfrac{1}{4\sqrt[4]{x^3}}dx \rightarrow dx = 4t^3dt}$

180. $\displaystyle\int \frac{dx}{\sqrt{x} - \sqrt[3]{x}}$

Solución:

Se aplica el método de sustitución o cambio de variable.

$$\sqrt[6]{x} = t$$
$$x = t^6$$
$$dx = 6t^5\,dt$$

Se obtiene:

$$2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\,L\,|\sqrt[6]{x} - 1| + k$$